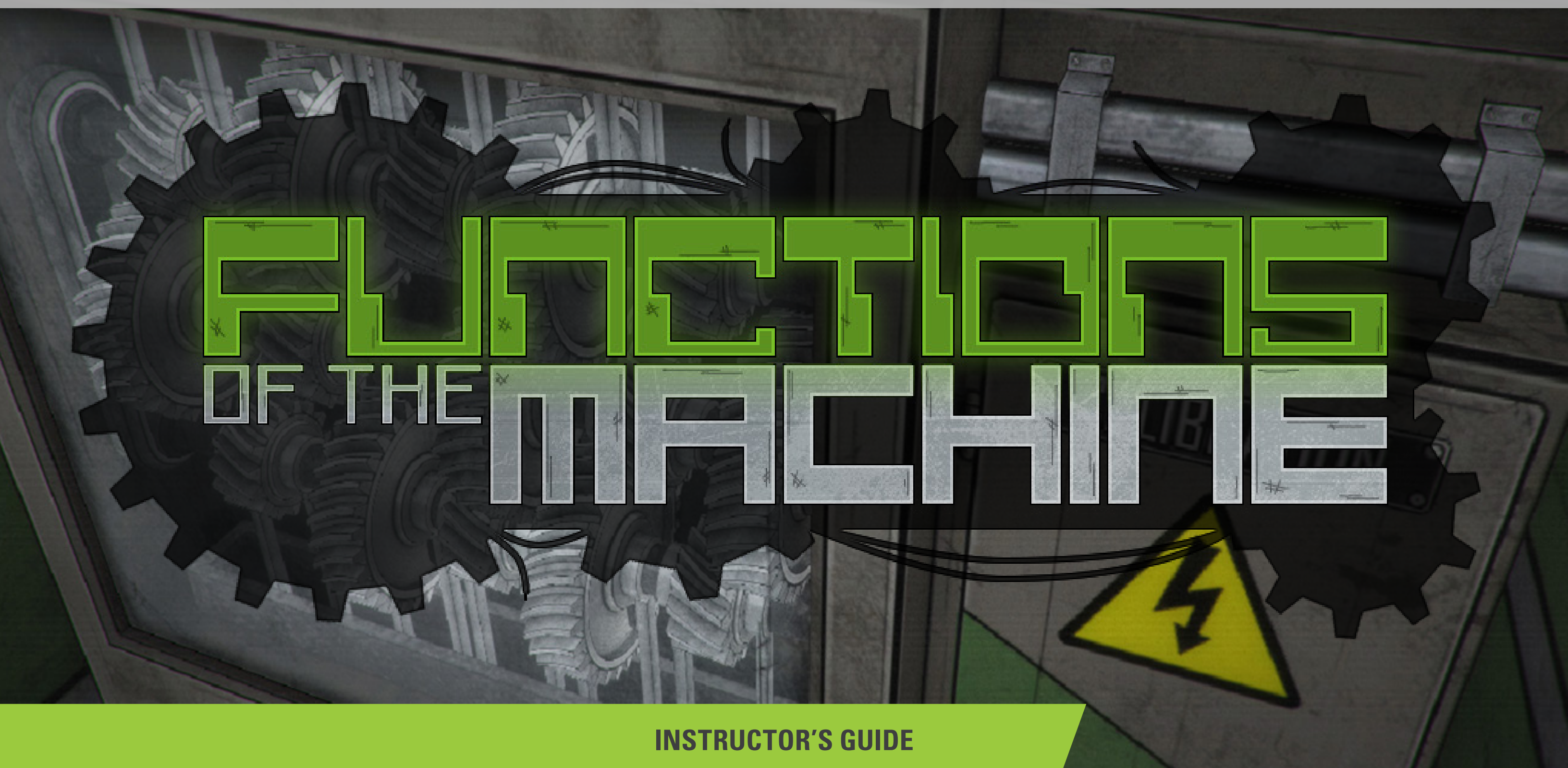


VIRTUAL LEARNING
EXPERIENCES

The K20 Center for Educational and Community Renewal is a statewide education research and development center which promotes innovative learning through school-university-community collaborations. Our mission is to cultivate a collaborative network engaged in research and outreach that creates and sustains innovation and transformation through leadership development, shared learning, and authentic technology integration.

The K20 Center's **Virtual Learning Experiences (VLE)** development team is tasked with creating game-based learning experiences to be used in undergraduate courses at The University of Oklahoma. The experiences are designed and developed by a small team working with volunteer University professors.

The purpose of this guide is to support the effective integration of Functions of the Machine into your classroom teaching. This guide provides an overview of the game and instructions for how students will access the game in your Canvas classroom.

TABLE OF CONTENTS

4 About the Game

5 Learning Goals

9 Playing the Game

11 Answer Key

12 References

13 Contacts

ABOUT THE GAME

PURPOSE

Researchers and subject matter experts have identified the **conceptual understanding of functions and how they relate to real-world phenomena** as topics of particular difficulty for students. Students often tend to think of functions in terms of a single algebraic formula that must be solved through computation, rather than a dynamic system of changing quantities that must be looked at more as a process (Carlson & Thompson, 2008). Additionally, students often have difficulty identifying the measurable attributes of a real-world situation and connecting those attributes with the graph of a function that represents that situation.

Functions of the Machine is intended for use in any algebra class that includes a unit on functions. This game helps students foster an understanding of **proportional reasoning, covariational reasoning, and a process view of functions.**

GAME NARRATIVE

> ***You've been charged with a peculiar task: decipher the function of a strange machine designed by an eccentric inventor. As an innovator in the field of manufacturing, Nicole Edisla has left a warehouse full of her machines in the wake of her auspicious disappearance. The purpose of these machines is a mystery, but Edisla has left extensive notes. It is up to you to use her notes and an understanding of functions to get the machine up and running once more.***

Functions of the Machine builds an understanding of the purpose and nature of mathematical functions though scaffolded problems. The Edisla machines represent functions–having a covariational relationship between their inputs and outputs. By solving problems with these machines, the student builds a conceptual understanding of functions. Each problem is more complex than the last, allowing students to slowly build their understanding as they progress through the game.

HOW TO USE THIS GAME

This virtual learning experience is offered as a supplement to traditional classroom instruction. We recommend having students play Functions of the Machine immediately before or after your initial lecture on the topic of functions. Functions of the Machine makes a good alternative homework assignment, extra credit assignment, or classroom group activity.

VLE games are accessed over the web and integrate with your learning management system, such as D2L or Canvas. VLE games can also be accessed through the K20 Game Portal website. Contact the K20 Center for help integrating this VLE into your classroom.

LEARNING GOALS

OBJECTIVES

- **Covariational Reasoning**
  The student can coordinate two varying quantities that change in tandem to solve a problem.

- **Process View of Functions**
  The student can explain that a function is not a set rule that defines a procedure but instead is a generalized input-output process.

- **Graphical Reasoning**
  The student can identify attributes of a graph that give meaning to the associated function's behavior.

- **Quantitative Reasoning**
  The student can identify and relate measurable attributes of an object or situation in a problem context.

COVARIATIONAL REASONING

Covariational reasoning has been defined as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). Carlson and Thompson defined a series of "mental actions" that classify the behaviors of students as they attempt to solve problems involving covariational reasoning.

Table 1 - Mental Actions

Mental Action	Description	Behavior
MA1	Coordinating the value of one variable with changes in the other.	Labeling the axes with verbal indications of coordinating the two variables.
MA2	Coordinating the direction of change of one variable with changes in the other variable.	• Constructing an increasing straight line. • Verbalizing an awareness of the direction of change of the output while considering changes in the input.
MA3	Coordinating the amount of change of one variable with the changes in the other.	• Plotting points/constructing secant lines. • Verbalizing an awareness of the amount of change in the output while considering changes in the input.
MA4	Coordinating the average rate of change of the function with uniform increments of change in the input variables.	• Constructing contiguous secant lines for the domain. • Verbalizing an awareness of the rate of change of the output (with respect to the input) while considering uniform increments of the input.
MA5	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function.	• Constructing a smooth curve with clear indications of concavity changes. • Verbalizing an awareness of the instantaneous changes in the rate of change for the entire domain of the function (direction of concavities and inflection points are correct).

(Carlson & Thompson, 2008, p. 14)

PROCESS VIEW OF FUNCTIONS

One method for improving students' understanding of functions is to foster their ability to employ covariational reasoning and to move away from viewing functions as memorized, static procedures. Students need to have a process view of functions, meaning that a function is not a set rule that defines a procedure but instead is a generalized input-output process (Carlson & Thompson, 2008). Put another way, the goal is to move the student from an *action view* of functions (a set of predefined steps) to a *process view* (a more global or system view).

Table 2 - Action and Process View of Functions

Action View	Process View
A function is tied to a specific rule, formula, or computation and requires the completion of specific computation and/or steps.	A function is a generalized input-output process that defines a mapping of a set of input values to a set of output values.
A student must perform or imagine each action.	A student can imagine the entire process without having to perform each action.
The "answer" depends on the formula.	The process is independent of the formula.
A student can imagine only a single value at a time as input or output (e.g., x stands for a specific number).	A student can imagine all input at once or "run through" a continuum of inputs. A function is a transformation of entire spaces.
Composition is substituting a formula or expression of x.	Composition is a coordination of two input-output processes; input is processed by one function and its output is processed by a second function.
Inverse is about algebra (switch y and x then solve) or geometry (reflect across y=x).	Inverse is the reversal of a process that defines a mapping from a set of output values to a set of input values.
Finding domain and range is conceived at most as an algebra problem (e.g., the denominator cannot be zero and the radicand cannot be negative).	Domain and range are produced by operating and reflecting on the set of all possible inputs and outputs.
Functions are conceived as static.	Functions are conceived as dynamic.
A function's graph is a geometric figure.	A function's graph defines a specific mapping of a set of input values to a set of output values.

(Carlson & Thompson, 2008, p. 10)

Carlson offers suggestions for the types of questions to ask for promoting a process view of functions and improving covariational reasoning abilities (Carlson & Thompson, 2008).

- Process view of functions:
  - Ask students to explain basic function facts in terms of input and output.
  - Ask about the behavior of functions on entire intervals in addition to single points
  - Ask students to make and compare judgments about functions across multiple representations.

- Covariational reasoning abilities:
  - Ask questions associated with each of the mental actions.
  - Ask for clarification of rate of change information in various contexts and representations.

QUANTITATIVE REASONING

Students also show a weakness in the area of quantitative reasoning, or the ability to identify and conceptualize the measurable attributes of an object or situation (Madison, 2015). This inability to identify the variables makes covariational reasoning even more difficult and is mentioned as another foundational concept by Carlson Jacobs, et al. (2002). As with covariational reasoning, Carlson, Jacobs, et al. (2002) provide the following recommendations for fostering better quantitative reasoning:

- What are the quantities in this situation? Which quantities are constant and which quantities are varying? Which varying quantities are you being asked to relate?

- As one quantity increases, how do the other quantities change? Can you define one variable to represent the varying value of one quantity and then use formulas to show how to calculate values of the other quantities?

- What does it mean to express one quantity "in terms of" another quantity?

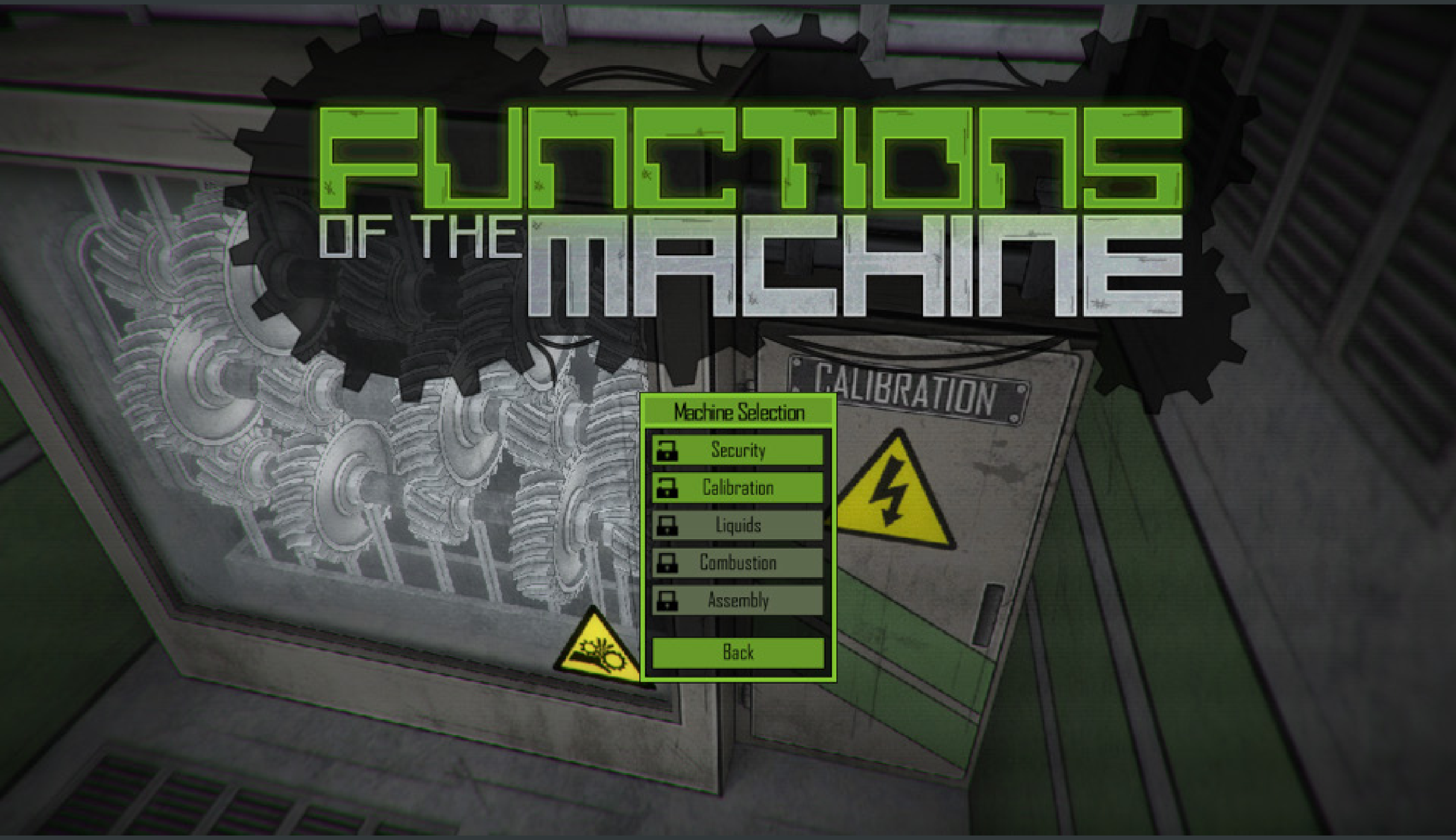
- Can you illustrate the relevant quantities in the situation in a drawing? Drawings are static and represent only one possible configuration of the situation; how else could a diagram of this situation look? What is the range of possibilities? What does the range imply about the possible values of the variables you identified?

- Can you explain in words how to determine values of one quantity when values of another quantity are known?

- What expression describes how to determine values of one quantity when values of another quantity are known?

PLAYING THE GAME

Functions of the Machine contains five levels, each of which contains multiple machines (problems). Levels are scaffolded so that the difficulty of each problem increases as the student proceeds through the game.



Each machine provides students with a unique covariational reasoning problem. Tutorials guide students through the operation of each machine.

Students cannot fail these tasks. If students enter the wrong value into a machine, corrective feedback helps direct their learning and they are allowed to try again. Students can proceed once they have selected the correct answer.

Some machines have multiple tasks. As students complete tasks, the level progress meter in the lower left corner of the screen fills.

The “Machine Control” button in the lower right corner of the screen opens a menu with multiple options.

From this menu, the student can adjust the volume, return to the main menu, or return to previous machines to play them again.

ANSWER KEY

Security 1, Combination Lock
- Task 1 – 5
- Task 2 – 15
- Task 3 – 10

Security 2, Crank Wheel
- Task 1 – ½
- Task 2 – ¾
- Task 3 – ¼

Security 1, Gear Lock
- Task 1 – 1
- Task 2 – ¼ or 5/4
- Task 3 – 3/2

Calibration 1, Gears of Different Speeds
- Task 1 – 9/6
- Task 2 – 4/6

Calibration 2, Gears of Different Sizes
- Task 1 – 3/2 Counterclockwise

Calibration 3, Bevel Gear
- Task 1 – 20
- Task 2 – 10
- Task 3 – 30

Calibration 4, Three Gears
- Task 1 – 12 Counterclockwise
- Task 2 – 8 Counterclockwise
- Task 3 – 4 Clockwise

Liquids 1, Lubrication
- Task 1 – Cylinder 2 = 20, Cylinder 3 = 5

Liquids 2, Fuel
- Task 1 – Cone 1 = 7/8, Cone 2 = 1/8

Liquids 3, Fuel Intake Valves
- Task 1 – Graph 3, Graph 1

Combustion 1, Fuel Intake
- Task 1 – 0.7, Graph 2

Combustion 2, Combustion Chamber
- Task 1 – 0.7, Graph 2

Combustion 3, Further Calibration
- Task 1 - 0.4, Graph 1

Assembly 1, Inflection Points
- Task 1 – B, F
- Task 2 – B, D, F

Assembly 2, Furthest Distance
- Task 1 – G

Assembly 3, Displacement
- Task 1 – Graph 3

Assembly 4, Pickup
- Task 1 – Graph 2
- Task 2 – Graph 1

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