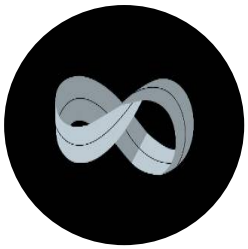

Things to Do With a Strip of Paper: Mobius Bands

3D Objects, Sides, Surfaces, and Edges

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Grade Level	6th – 10th Grade	**Time Frame**	2-3 class period(s)
Subject	Mathematics	**Duration**	100 minutes
Course	Geometry, Middle School Mathematics		

Essential Question

What is a Mobius band, and what can I do with it?

Summary

The main purpose of this lesson and others in the series "Things to Do With a Strip of Paper" is to explore interesting mathematical and topological structures that students of any age can enjoy. This lesson focuses on the Mobius band (or Mobius strip) and provides students with opportunities to better understand its unique properties and how they connect to mathematics. Students will explore shapes, symmetries, and sides.

Snapshot

Engage

Students watch a video about Wind and Mr. Ug and make conjectures about what is happening in the story.

Explore

Students explore curiosities about their Mobius bands.

Explain

Students discuss their ideas and articulate the peculiarities of their Mobius bands.

Extend

Students extend their learning by cutting Mobius bands in three different ways.

Evaluate

Students reflect on their learning by completing a What? So What? Now What? handout.

Standards

Oklahoma Academic Standards for Mathematics (Process Standards)

M.6: Develop the Ability to Make Conjectures, Model, and Generalize: Make predictions and conjectures and draw conclusions throughout the problem solving process based on patterns and the repeated structures in mathematics. Students will create, identify, and extend patterns as a strategy for solving and making sense of problems.

Attachments

- Lesson-Slides-Mobius-Bands.pptx
- Mobius-Bands-Strips-Things-To-Do-with-a-Strip-of-Paper - Spanish.docx
- Mobius-Bands-Strips-Things-To-Do-with-a-Strip-of-Paper - Spanish.pdf
- Mobius-Bands-Strips-Things-To-Do-with-a-Strip-of-Paper.docx
- Mobius-Bands-Strips-Things-To-Do-with-a-Strip-of-Paper.pdf
- More-About-Mobius-Mobius-Bands - Spanish.docx
- More-About-Mobius-Mobius-Bands - Spanish.pdf
- More-About-Mobius-Mobius-Bands.docx
- More-About-Mobius-Mobius-Bands.pdf
- The-Mobius-Band-Effect-Mobius-Bands - Spanish.docx
- The-Mobius-Band-Effect-Mobius-Bands - Spanish.pdf
- The-Mobius-Band-Effect-Mobius-Bands.docx
- The-Mobius-Band-Effect-Mobius-Bands.pdf
- What-So-What-Now-What-Mobius-Bands - Spanish.docx
- What-So-What-Now-What-Mobius-Bands - Spanish.pdf
- What-So-What-Now-What-Mobius-Bands.docx
- What-So-What-Now-What-Mobius-Bands.pdf

Materials

- Mobius Bands handout (attached; one per student), or copy paper cut lengthwise into approximately 2" x 11" strips (four strips per student)
- The Mobius Band Effect handout (attached; one per pair of students)
- More About Mobius handout (attached; one per pair of students)
- What? So What? Now What? handout (attached; one per student)
- Lesson Slides (attached)
- Pens or pencils
- Tape or glue
- Rulers

Engage

Teacher's Note: Programming Alert

In order to not spoil the exploration, make sure to stop the video at 6:45!

Display **slides 3-4** to review the essential question and lesson objectives to the extent you feel is necessary. Go to **slide 5**. Show students the Wind and Mr. Ug video.

Teacher's Note: What Is Happening?

The world of Wind and Mr. Ug is actually a transparent Mobius band, but students might not realize this right away. The next activity asks students to collaborate with their peers to see if they can figure out what is happening.

Go to **slide 6**. Ask students to engage in a Think-Pair-Share activity:

1. Think: Ask students to think to themselves about what might be happening in the video.
2. Pair: Ask students to find a partner. Pairs then discuss what they think is happening.
3. Share: After students discuss with a partner, solicit responses from the whole class.

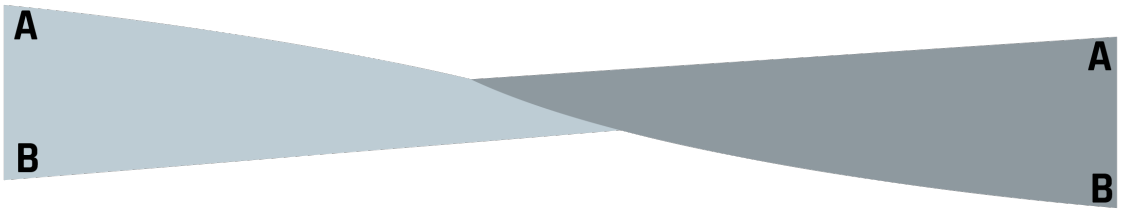
Play the remainder of the video, and share with students that they are going to build a world like Wind and Mr. Ug's.

Explore

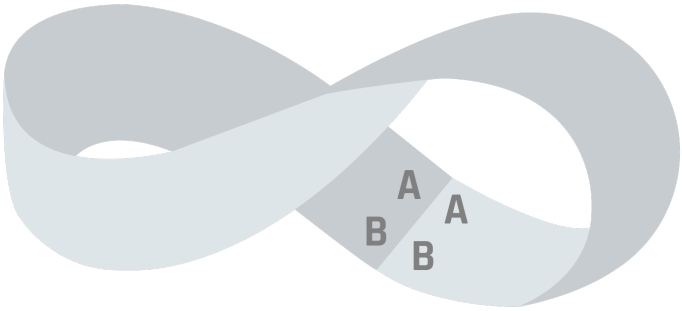
Students will now make their own Mobius bands. Each student needs a strip of paper, tape or glue, a pen, and scissors. Pre-labeled **Mobius Band Strips** are attached if you prefer to save steps and give each student an already cut and labeled strip of paper.

Go to **slide 7**. To make their Mobius bands, students will first label the strip of paper and then twist it 180 degrees. After the twist, they will match up the letters and then tape or glue the ends together. Diagrams are included below and in the lesson slides to assist with construction.

Label the ends of a cut strip of paper AB and BA (pictured above).



Twist one end of the paper strip so that it looks like this.



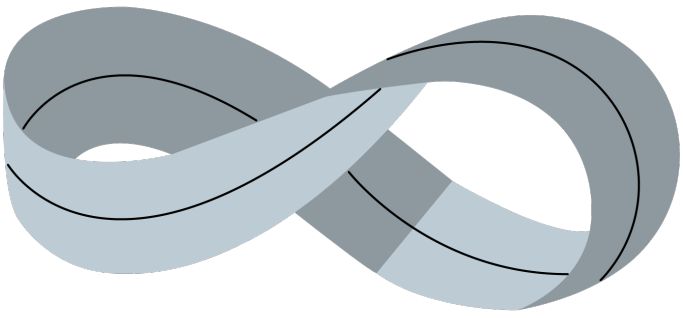
Connect the letters (A matches to A and B matches to B).

Teacher's Note: Cylindrical Loop Or Mobius Band

Check students' Mobius bands to make sure they joined A to A and B to B. If they join A to B and B to A, they will create a cylindrical loop. There's nothing wrong with this, but it isn't a Mobius band. The twist is essential!

Tell students, "We now need to figure out what is happening in the world of Wind and Mr. Ug."

Go to **slide 8**. To begin thinking about this, instruct students to draw a line down the center of their Mobius bands as illustrated below.



A completed Mobius band with a line drawn down the middle

Ask the following questions to guide students as they explore their Mobius bands. Ask students to work with a partner to answer the questions.

1. How many lines did you draw around your Mobius band?
2. How many sides does your Mobius band have?
3. How many edges does your Mobius band have?
4. Does this help explain the world of Wind and Mr. Ug?

Sample Student Responses

The students should respond that they drew one line, which means that the Mobius band has only one side and one edge, but this may not be a response they arrive at until they explain their responses in the next portion of the lesson. Some students will figure out that the world of Wind and Mr. Ug is "continuous."

Explain

Go to **slide 9**. Give partners **The Mobius Band Effect handout**. Have students work with their partners to discuss the questions to help clarify misconceptions that they might have about their Mobius bands.

1. Describe the world of Wind and Mr. Ug to someone who might not understand. What is happening? Why is this interesting?
2. Can you prove that your Mobius band has only one side and only one edge? Explain how.
3. Is the Mobius band a 2D or a 3D object? How do you know?

Let students come to a consensus, and then solicit a few answers from the class.

Sample Student Responses

The students should arrive at the conclusion that the Mobius band is a one-sided, one-edged, three-dimensional object.

Go to **slide 10**. Share the background information about Mobius bands with students.

Extend

In this part of the lesson, students will further explore the Mobius band with their partners and predict what they think will happen when cutting it in different ways.

Extension Activity 1

Go to **slide 11**. For this activity, students need their Mobius band from the Explore activity. It should have a line drawn down the center.

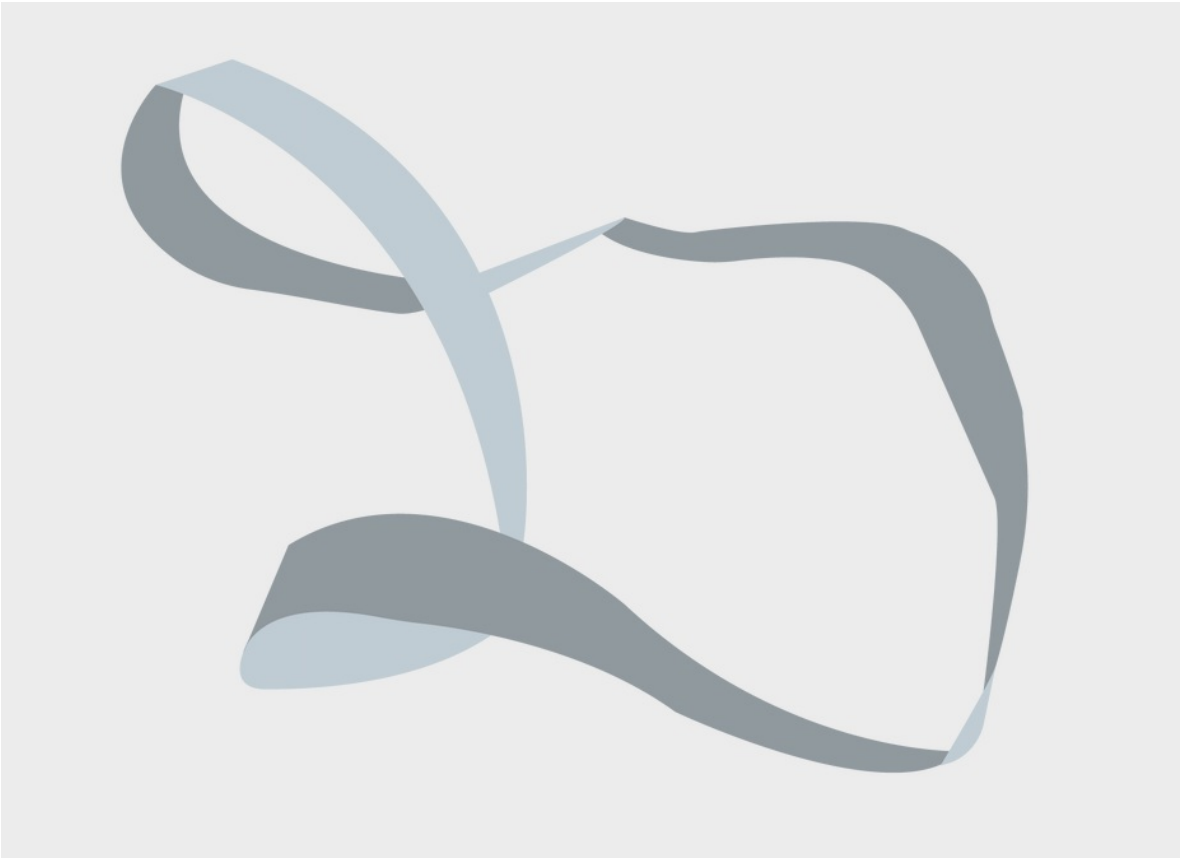
Pass out copies of the **More About Mobius handout**. Pose the following question, and ask students to write down their predictions: What will happen when we cut along the line?

Instruct students to cut along the line, all the way around.

Ask students to write down their answers to the following questions on their More About Mobius handouts:

1. What happened? Describe the result.
2. How did this compare to your prediction?
3. Is the result a Mobius band? How can you tell?

Go to **slide 12** to reveal the result of Extension Activity 1.



Result of Extension Activity 1: A twisted loop, but not a Mobius band

Extension Activity 2

Go to **slide 13**. Students will need to create a new Mobius band to use for this activity.

Pose the following question, and ask students to write down their predictions: What will happen when we cut one third from the edge all the way around?

This time, ask students to cut 1/3 from the edge instead of down the middle. Students should complete the cut all the way around.

Ask students to discuss and write down answers to the following questions:

1. What happened this time? Describe the result.
2. Did this match your prediction?
3. Are both of these Mobius bands? How can you tell?

Go to **slide 14**. Ask students to discuss and write down answers to the following questions:

1. What do you think would happen if we cut 1/4 from the edge?
2. How about 1/5?

Next, ask students: How much longer do you think the larger loop is when compared to the smaller one? Jot down your prediction, and measure. Students will need to un-tape, unglue, or cut their Mobius bands, and then measure.

Sample Student Response

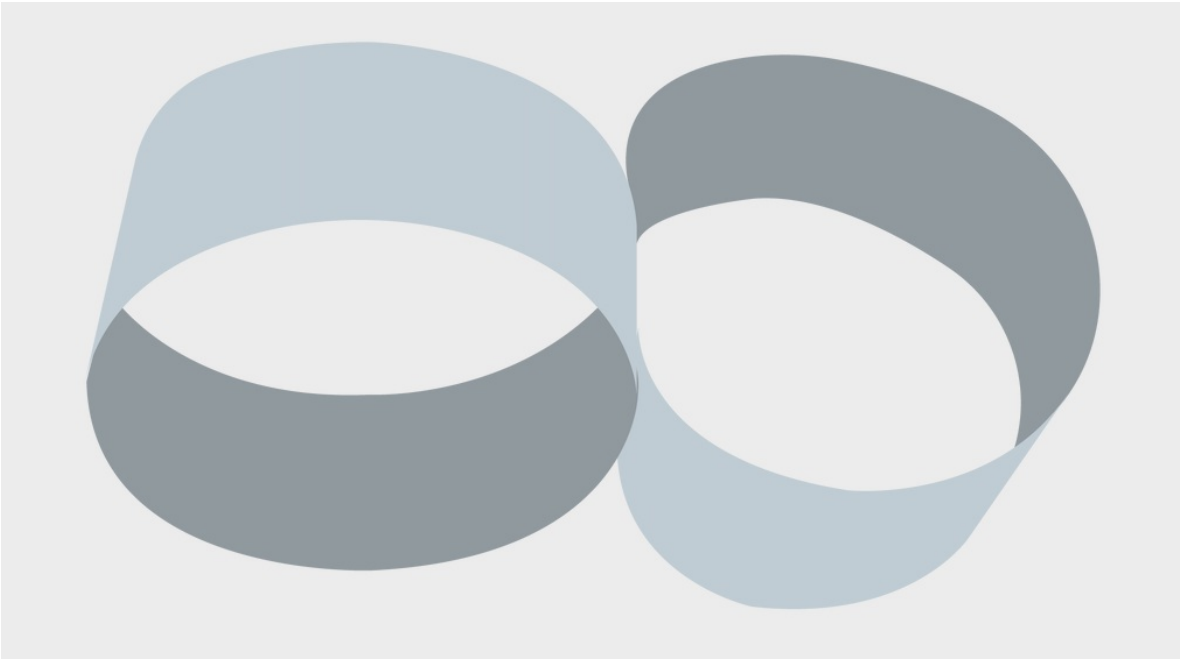
The larger loop should be about twice the length of the smaller loop.

Go to **slide 15** to reveal the result of Extension Activity 2.

Result of Extension Activity 2: Two loops; one is a twisted loop, as in the picture above, with a Mobius band linked

Extension Activity 3

Go to **slide 16**. Pass out two more strips of paper to each student. This time, instruct students to form each into a cylindrical loop and then tape the two loops together at a right angle as depicted below.



Two cylindrical loops taped together at a right angle

Go to **slide 17**. Pose the following question, and ask students to write down their predictions: What will happen when we cut down the center of each loop all the way around?"

Now, instruct students to cut down the center of each loop. Ask students: What happened? How does your result compare with what you predicted?

Go to **slide 18** to reveal the result of Extension Activity 3.

Result of Extension Activity 3: A square

Evaluate

Go to **slide 19**, and pass out copies of the **What? So What? Now What? handout**. Ask students to respond to the What? So What? Now What? prompts to assess their learning.

- **What?** What did you do in this lesson?
- **So what?** What did you learn that you did not know before?
- **Now what?** Where do you think you might use Mobius bands in your everyday life?

Resources

- Hart, V. (2015, December 4). Wind and Mr. Ug [Video]. Vimeo. https://vimeo.com/147906386
- K20 Center. (n.d.). Think-Pair-Share. Strategies. https://learn.k20center.ou.edu/strategy/139
- K20 Center. (n.d.). What? So What? Now What? Strategies. https://learn.k20center.ou.edu/strategy/95
- Lawrence, S. (2005). Mobius strip. Maths is Good for You! http://www.mathsisgoodforyou.com/worksheets/mobiustrip
- Newton, A. (n.d.). The Mobius strip. http://jwilson.coe.uga.edu/EMAT6680Su10/Newton/emat6690/MobiusStrips/MobiusStrips.html