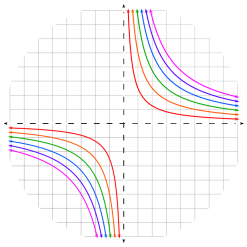

Can't Touch This, Part 2

Graphing Rational Functions

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Grade Level	10th – 11th Grade	**Time Frame**	90-120 minutes
Subject	Mathematics	**Duration**	2-3 periods
Course	Algebra 2		

Essential Question

What can cause asymptotes?

Summary

In this lesson, students will work with rational functions that have 0, 1, or 2 vertical asymptotes and 0 or 1 horizontal asymptotes. Students will explore the relationship between the equation and graph of a rational function, learn what causes different types of asymptotes, and apply their knowledge to graph rational functions. This is the second lesson of two in the "Can't Touch This" lesson series—see "Can't Touch This, Part 1" for prerequisite content.

Snapshot

Engage

Students organize graphs of rational functions that have 0, 1, or 2 vertical asymptotes and 0 or 1 horizontal asymptotes into groups through a Card Sort activity.

Explore

Students explore the relationship between the equation and the graph of a rational function.

Explain

Students complete guided notes with the class and formalize their understanding of graphing rational functions.

Extend

Students apply what they have learned to graph rational functions.

Evaluate

Students match graphs of rational functions with asymptotes and equations in a Card Matching activity.

Standards

Oklahoma Academic Standards for Mathematics (Grades 9, 10, 11, 12)

A2.F.1.6: Graph a rational function and identify the x- and y-intercepts, vertical and horizontal asymptotes, using various methods and tools that may include a graphing calculator or other appropriate technology. (Excluding slant or oblique asymptotes and holes.)

Attachments

- Card-Matching-Can-t-Touch-This-Part-2.pdf
- Card-Sort-Can-t-Touch-This-Part-2.pdf
- Exploring-Rational-Functions-Can-t-Touch-This-Part-2 - Spanish.docx
- Exploring-Rational-Functions-Can-t-Touch-This-Part-2 - Spanish.pdf
- Exploring-Rational-Functions-Can-t-Touch-This-Part-2.docx
- Exploring-Rational-Functions-Can-t-Touch-This-Part-2.pdf
- Graphing-With-Asymptotes-Part-2-Can-t-Touch-This-Part-2 - Spanish.docx
- Graphing-With-Asymptotes-Part-2-Can-t-Touch-This-Part-2 - Spanish.pdf
- Graphing-With-Asymptotes-Part-2-Can-t-Touch-This-Part-2.docx
- Graphing-With-Asymptotes-Part-2-Can-t-Touch-This-Part-2.pdf
- Guided-Notes-Can-t-Touch-This-Part-2 - Spanish.docx
- Guided-Notes-Can-t-Touch-This-Part-2 - Spanish.pdf
- Guided-Notes-Can-t-Touch-This-Part-2.docx
- Guided-Notes-Can-t-Touch-This-Part-2.pdf
- Guided-Notes-Teacher-Guide-and-Model-Notes-Can-t-Touch-This-Part-2.pdf
- Lesson-Slides-Can-t-Touch-This-Part-2.pptx

Materials

- Lesson Slides (attached)
- Card Sort document (attached; one set per pair; printed front only)
- Exploring Rational Functions handout (attached; one per pair; printed front only)
- Guided Notes handout (attached; one per student; printed front/back)
- Graphing With Asymptotes, Part 2 handout (attached; one per student; printed front/back)
- Card Matching document (attached; one set per pair; printed front only)
- Pencils
- Scientific calculators
- Student devices with internet access
- Guided Notes (Teacher Guide and Model Notes) (optional; attached)

10 minutes

Engage

Teacher's Note: Preparation

Before you begin, print the attached **Card Sort** and **Card Matching** documents (one copy per pair of students in your class). Consider printing the graph cards from the Card Sort document in color on white paper; consider also printing the remaining cards from the Card Matching document in black and white, with each page printed on a different color of paper. For example, you might print all asymptote cards (page 1) on orange paper and all equation cards (page 2) on yellow paper. Additionally, consider printing on cardstock paper, especially if you plan to reuse these cards.

Once printed, cut out the cards. All of these cards are the same size for easy cutting.

The cards from the Card Sort document will be used during both the Engage and Evaluate phases of this lesson, while the cards from the Card Matching document will only be used during the Evaluate phase.

Introduce the lesson using the attached **Lesson Slides**. Display **slide 3** to show the lesson's essential question: *What can cause asymptotes?* **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Ask students to find a partner or assign partners yourself. Remind students to be kind and careful with the printed cards, then pass out one set of graph cards from the **Card Sort** document to each pair of students. Show **slide 5**. Share the instructional strategy Card Sort with the class, and have students use this strategy to group the nine graph cards into two, three, or four groups of their choosing. After students have had a chance to organize their cards into groups, ask students to find another pair with which to discuss their thinking.

As time allows, ask for volunteers to share with the class how they chose to organize their cards or how they would describe their groups of cards.

Optional: Alternative Structure

Consider having each student get out a paper and pencil before they start their Card Sort. Have students divide their paper into fourths, labeling the spaces "Group 1," "Group 2," "Group 3," and "Group 4."

Next, have students sort their cards into two, three, or four groups as described above. Then ask students to write a description for each card group they created in the appropriate space on their paper. This can help students organize their thoughts before sharing with another pair of students and/or with the whole class.

Collect each set of graph cards. These will be used later in the lesson (likely on a different day of class).

20 minutes

Explore

Display **slide 6**. On your Desmos Dashboard, press the orange plus sign to allow students to progress to **screen 5.** Direct students to read the directions on their screen and to select the GeoGebra activity link: geogebra.org/m/d9ywzkrc. This interactive GeoGebra activity includes two GeoGebra applets: the first is focused on vertical asymptotes, while the second is focused on horizontal and slant asymptotes. Invite students to interact with both. This gives students a chance to explore rational functions that do not always have one vertical and one horizontal asymptote as they saw in the previous lesson, "Can't Touch This, Part 1."

After giving students time to explore the GeoGebra activity, pass out one of the attached **Exploring Rational Functions** handouts to each pair of students. Instruct students to work with their partner to complete the handout. In each applet, there is a reset button in the top-right corner that looks like two arrows making a circle. Encourage students to use this button if their exploration makes the graph difficult to see.

Sample Student Responses

As students explore vertical asymptotes, they may observe the following:

- When m and n are both 1, b_0 shifts the vertical asymptote left and right.
- When m and n are both 2 and b_0 is positive, there are two vertical asymptotes. The graph is broken into three pieces.
- When m and n are both 2 and b_0 is negative, there are zero vertical asymptotes. The graph looks like a bump.

As students explore horizontal and slant asymptotes, they may observe the following:

- When n = 1 and m = 0 or 1, the graph looks like what we saw in the previous lesson—one vertical and one horizontal asymptote.
- When n = 1 and m = 2, the horizontal asymptote is not horizontal—maybe it's a slant asymptote. When the graph is zoomed out, it looks like some of the work done in part 1 of this lesson, but the asymptote is a line with a positive slope.
- When n = 2 and m = 0 or 2, there are two vertical asymptotes.
- When n = 2 and m = 1 or 3, the curve crosses an asymptote—one was horizontal and one wasn't.
- When n = 1 and m = 3, the graph looks weird, like a non-linear asymptote and a vertical asymptote.

After students complete the GeoGebra activity, ask for volunteers to share their observations with the class.

Teacher's Note: Guiding the Activity

Students may make observations that significantly differ from the sample responses above. Use their responses to check for misunderstandings. Encourage students to do their best to use academic language to describe what they see.

Regarding the non-linear asymptote, students are not expected to find the equations of non-linear asymptotes or graph equations with non-linear asymptotes in high school. The purpose here is exposure and to emphasize that slant asymptotes exist when the degree of the numerator is only one greater than the degree of the denominator. Students are often expected to write the equations for and graph slant asymptotes in precalculus courses. In Algebra 2, students only need to recognize that they exist and that not every rational function has a horizontal asymptote.

25 minutes

Explain

Go to **slide 7**. Give each student a copy of the attached **Guided Notes** handout. Distribute a scientific calculator to each student. Complete the handout as a class. Students should use the table feature of their scientific calculators to save time when calculating y-values.

Teacher's Note: Guiding the Activity

When graphing the second example on the handout, consider using a yardstick as a visual example of the slant asymptote. Explain to students that they do not need to draw it on the graph since they did not calculate its exact slope and y-intercept—instead, use the yardstick to help students imagine where the slant asymptote would be.

For more support and recommendations for the Guided Notes, use the attached **Guided Notes (Teacher Guide and Model Notes)** document.

Sample Questions & Responses

Students are likely to ask why the curve can cross a horizontal or slant asymptote. The curve is allowed to cross horizontal and slant asymptotes because those asymptotes give guidance to the ends of the graph. In other words, a horizontal or slant asymptote describes what the graph is doing at its very-far-left end (as x approaches negative infinity) and at its very-far-right end (as x approaches positive infinity). Those asymptotes *do not* describe what the graph is approaching near the center of the graph, like when x = 0.

Once finished, have students add the handout to their math notebook if that is a classroom norm.

25 minutes

Extend

Have each pair of students find another pair and partner up to create groups of four students. Pass out a copy of the attached **Graphing With Asymptotes, Part 2** handout to each student and display **slide 8**. Instruct students to work with their group to graph the first rational function.

As groups finish question 1, transition through **slides 9–10** so students can check their work. Bring the class together for questions and discussion. Ask for volunteers to explain why question 1 does not have any vertical asymptotes. Ask students to explain how they determined the horizontal asymptote. Use student responses to help clarify any misunderstandings.

Instruct students to now work with only one person from their group of four to complete question 2.

As students work through and complete question 2, transition through **slides 11–12** so students can check their work. Bring the class together once again for questions and discussion. Ask for volunteers to explain why question 2 has two vertical asymptotes. Ask students to explain how they determined the horizontal asymptote. Ask students if the curve crossed or touched an asymptote and why it is possible for a curve to touch a horizontal asymptote. Use student responses to help clarify any misunderstandings.

Challenge students to now work independently to complete question 3. Remind students that this is a great opportunity to reflect on what they know and what questions they may have.

As students complete question 3, transition through **slides 13–14** so students can check their work. Give students an opportunity to ask questions and correct misunderstandings.

10 minutes

Evaluate

Teacher's Note: Preparation

Before starting the next activity, decide whether you want the below Card Matching activity to be guided practice or independent practice. To have students check their own work, unhide slide 16 from view before you begin. Alternatively, you can print slide 16 and use it as a key to check the matches on students' desks.

Direct each student to find their original partner. Show **slide 15**. Return to each pair a set of graph cards from the Engage phase Card Sort activity. Pass out the asymptotes and equation cards from the **Card Matching** document as well. Direct students to complete their Card Matching activity by matching each graph card with an asymptotes card and an equation card together. In other words, students should end up with several groupings of three cards. Remind students that there are six sets of cards, so three of the graphs will not have matching asymptotes or equation cards. These three cards should not be matched together.

While students are working, walk around the classroom and ask students to share their thinking about certain matches you see they've made. Remember to encourage academic vocabulary. While circulating the room, use what you hear students talking about and what questions they ask to see what misconceptions still exist.

Optional: Slide 16

If desired, show **slide 16** to allow students to compare their work with the final results. Ask for volunteers to explain their process.

At the end of the class period, remember to collect students' cards and prepare each set for the next class if necessary.

Resources

- K20 Center. (n.d.). Card Matching. Strategies. https://learn.k20center.ou.edu/strategy/1837
- K20 Center. (n.d.). Card Sort. Strategies. https://learn.k20center.ou.edu/strategy/147
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081
- K20 Center. (n.d.). GeoGebra. Tech tools. https://learn.k20center.ou.edu/tech-tool/2352