GUIDED NOTES (TEACHER GUIDE)

<u>**Change of Base**</u>: $\log_b a = \dfrac{\log a}{\log b} = \dfrac{\ln a}{\ln b}$

Teacher	Students
The logarithm with base b of a must equal something; let's call that x.	$\log_b a = x$
Let's remove the logarithm with base b by using the exponential function with base b (exponentiate both sides using base b) and simplify. *Remind students that this is the same as rewriting the equation into exponential form.*	$b^{\log_b a} = b^x$ $a = b^x$
Let's take the "log base 10" of both sides.	$\log(a) = \log(b^x)$
Using the power property, let's move the exponent in front of the "log of b."	$\log(a) = x \cdot \log(b)$
Now, how would we solve for x?	Divide both sides by the "log of b." $\dfrac{\log(a)}{\log(b)} = \dfrac{x \cdot \log(b)}{\log(b)}$ $\dfrac{\log(a)}{\log(b)} = x$
Let's rewrite it as "x equals the log of a over the log of b."	$x = \dfrac{\log(a)}{\log(b)}$
Look at the beginning. What did x equal?	the "log base b of a" $x = \log_b a$
Let's substitute that in for x. Now, we can see why the change of base formula works. We could have done this exact same process with a natural logarithm instead—or any logarithm, for that matter.	$\log_b a = \dfrac{\log(a)}{\log(b)}$ $\log_b a = \dfrac{\ln(a)}{\ln(b)}$

The change of base formula also can be written as $\log_m n = \dfrac{\log_b n}{\log_b m}$ *for any base* b*. However, the formula versions provided in the Guided Notes are typically the most useful, as they allow students to get a decimal approximation easily using a scientific calculator.*

Solve each of the following equations.

1) $\log_7(x+7)+\log_7(x+1)=1$

Teacher	Students
Here, we see two logarithms and a constant. To be able to solve using inverse operations, we would need to see a logarithm equaling a logarithm or be able to isolate a logarithm. So, which property of logarithms do you think we could use to put these two logarithms together?	product property
Using the product property, we now have the "log base 7" of the product of the quantities $x+7$ and $x+1$ equals 1.	$\log_7\left((x+7)(x+1)\right)=1$
Let's expand the binomials within the logarithm.	$\log_7\left(x^2+8x+7\right)=1$
If we want to remove the logarithm with a base of 7, what should we do?	Use an exponential function with a base of 7. $7^{\log_7\left(x^2+8x+7\right)}=7^1$ $x^2+8x+7=7$
Now, we have a quadratic equation. How do we solve for x?	Subtract 7 from both sides. $\begin{array}{r} x^2+8x+7=7 \\ -7\ \ -7 \\ \hline x^2+8x=0 \end{array}$

Teacher	Students
What should we do next?	Factor out the greatest common factor (GCF) and set each factor equal to zero to solve. $$x(x+8)=0$$ $$x=0 \text{ and } x+8=0$$ $$x=0 \text{ and } x=-8$$
Now, we see that $x=0$ and $x=-8$. Remember that a logarithm has a domain restriction. We can only take the logarithm of a positive value; the logarithm of a negative value or zero is undefined.	
To find the domain restriction(s), let's look at the original problem and set what is in the logarithm to be greater than zero, then solve for x. In this case, there are two quantities that need to be positive: $x+7$ and $x+1$.	$$\log_7(x+7)+\log_7(x+1)=1$$ $$x+7>0 \text{ and } x+1>0$$
How would we solve each of these for x?	Subtract 7 from both sides and subtract 1 from both sides. $$\begin{array}{ccc} x+7>0 & \text{and} & x+1>0 \\ \underline{-7\ \ -7} & & \underline{-1\ \ -1} \\ x>-7 & \text{and} & x>-1 \end{array}$$
Both statements must be true, so what set of numbers is both greater than negative 7 and greater than negative 1?	Numbers that are greater than negative 1. $$x>-1$$
So, our solution(s) need(s) to be greater than negative 1. Is $x=0$ or $x=-8$ greater than negative 1?	Zero is greater than negative 1.
This means that $x=0$ is the solution, and $x=-8$ is an extraneous solution.	$$x>-1$$ $$\boxed{x=0}$$

2) $\log_2\left(x^2+10\right)-\log_2\left(7\right)=1$

Teacher	Students
Here, we see two logarithms and a constant. Which property of logarithms do you think we could use to put these two logarithms together?	quotient property
Why would you suggest the quotient property?	because there is subtraction between the two logarithms
Using the product property, we now have the "log base 2" of the quotient of the quantity x^2+10 and 7 equals 1.	$\log_2\left(x^2+10\right)-\log_2\left(7\right)=1$ $\log_2\left(\frac{x^2+10}{7}\right)=1$
If we want to remove the logarithm with a base of 2, what should we do?	Use an exponential function with a base of 2. $2^{\log_2\left(\frac{x^2+10}{7}\right)}=2^1$ $\frac{x^2+10}{7}=2$
What is the next step to isolate x?	Multiply both sides by 7. $7\cdot\frac{x^2+10}{7}=2\cdot 7$ $x^2+10=14$
What should we do next?	Subtract 10 from both sides. $x^2+10=14$ $\underline{-10\ \ -10}$ $x^2=4$
How do we get x by itself?	Take the square root of both sides. $\sqrt{x^2}=\sqrt{4}$ $x=\pm 2$

Teacher	Students
We have now solved for x. But how do we ensure that x is allowed to be ± 2?	We need to check for domain restrictions.
How would we do that?	Look at the original problem and write what is inside the logarithm as greater than zero. $\log_2\left(x^2+10\right)-\log_2\left(7\right)=1$ $x^2+10>0$
Notice that we do not need to write $7>0$ since that is always true. We are checking for domain restrictions only—which means restrictions on x.	
How would we solve for x?	Subtract 10 from both sides. $x^2+10>0$ $\underline{\quad -10 \;\; -10}$ $x^2>-10$
x^2 is always greater than a negative number, so this inequality is always true. Therefore, there is no domain restriction. Both $x=-2$ and $x=2$ are the solutions.	*no restrictions* $\boxed{x=\pm 2}$

3) $\log_3(5-3x)=\log_3(4x-9)$

Teacher	Students
Try this example on your own. *As students work on their own, slowly write the steps of solving on the board so that students can check their work as they go.* *Make sure that students are ahead of what you are writing. Use this time to circulate the room and check in with students to clarify any misunderstandings.*	$\log_3(5-3x)=\log_3(4x-9)$ $3^{\log_3(5-3x)}=3^{\log_3(4x-9)}$ $5-3x=4x-9$ $14=7x$ $2=x$ $\textit{Domain Restriction}$ $5-3x>0$ and $4x-9>0$ $-3x>-5$ and $4x>9$ $x<\frac{5}{3}$ and $x>\frac{9}{4}$ $\boxed{\textit{no solution}}$
Students can tackle this problem in different ways. Some may start with the domain restriction so as not to forget that key step. Others may solve the equation first. Both approaches are great. If time allows, talk to students about the different approaches once they have finished.	
Depending on the approach, a student who found the domain restriction first could come to the conclusion of "no solution" without having to solve for x, as there is no number that is both less than $\frac{5}{3}$ *and greater than* $\frac{9}{4}$. *Other students may take their* $x=2$ *and check both inequalities before coming to the same conclusion: There is* ***no solution****.*	

GUIDED NOTES (MODEL NOTES)

Properties of Logarithms

Product Property: $\log_b(m \cdot n) = \log_b(m) + \log_b(n)$

Quotient Property: $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$

Power Property: $\log_b(m^p) = p \cdot \log_b(m)$

Change of Base: $\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$

Examples

Solve each of the following equations.

1) $\log_7(x+7) + \log_7(x+1) = 1$

$\log_7((x+7)(x+1)) = 1$

$\log_7(x^2+8x+7) = 1$

$7^{\log_7(x^2+8x+7)} = 7^1$

$x^2+8x+7 = 7$

$x^2+8x = 0$

$x(x+8) = 0$

$x = 0$ and $x+8 = 0$

$x = 0$ and $x = -8$

Domain Restriction

$x+7 > 0$ and $x+1 > 0$

$x > -7$ and $x > -1$

$x > -1$

$\boxed{x = 0}$

2) $\log_2(x^2+10) - \log_2(7) = 1$

$\log_2\left(\frac{x^2+10}{7}\right) = 1$

$2^{\log_2\left(\frac{x^2+10}{7}\right)} = 2^1$

$\frac{x^2+10}{7} = 2$

$x^2+10 = 14$

$x^2 = 4$

$\boxed{x = \pm 2}$

Domain Restriction

$x^2+10 > 0$

$x^2 > -10$ (always true)

no restrictions

3) $\log_3(5-3x) = \log_3(4x-9)$

$3^{\log_3(5-3x)} = 3^{\log_3(4x-9)}$

$5-3x = 4x-9$

$14 = 7x$

$2 = x$

Domain Restriction

$5-3x > 0$ and $4x-9 > 0$

$-3x > -5$ and $4x > 9$

$x < \frac{5}{3}$ and $x > \frac{9}{4}$

$\boxed{\textit{no solution}}$