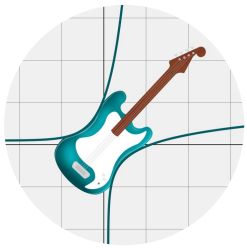

All About That Base, Part 2

Solving Logarithmic Equations

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Grade Level	10th – 11th Grade	**Time Frame**	80-90 minutes
Subject	Mathematics	**Duration**	2 class periods
Course	Algebra 2		

Essential Question

How do we use properties of logarithms and exponential functions to solve logarithmic equations?

Summary

In this lesson, students will discover properties of logarithms. Students will use the properties to work through a proof of the change of base formula, to solve logarithmic equations, and to evaluate logarithmic expressions. Prerequisites for this lesson include solving exponential equations using logarithms and understanding extraneous solutions and domain restrictions. This is the second lesson in the "All About That Base" lesson duo.

Snapshot

Engage

Students recall solving exponential equations using logarithms.

Explore

Students find patterns and discover properties of logarithms.

Explain

Students complete guided notes with the class and formalize their understanding of using properties of logarithms and solving logarithmic equations.

Extend

Students apply what they have learned to solve logarithmic equations and evaluate logarithmic expressions using properties of logarithms.

Evaluate

Students use the Justified True or False strategy to reflect on their learning and identify and resolve misconceptions.

Standards

Oklahoma Academic Standards for Mathematics (Grades 9, 10, 11, 12)

A2.A.1.6: Solve common and natural logarithmic equations using the properties of logarithms.
A2.F.2.4: Apply the inverse relationship between exponential and logarithmic functions to convert from one form to another.

Attachments

- Applying Properties—All About That Base, Part 2 - Spanish.docx
- Applying Properties—All About That Base, Part 2 - Spanish.pdf
- Applying Properties—All About That Base, Part 2.docx
- Applying Properties—All About That Base, Part 2.pdf
- Finding Patterns—All About That Base, Part 2 - Spanish.docx
- Finding Patterns—All About That Base, Part 2 - Spanish.pdf
- Finding Patterns—All About That Base, Part 2.docx
- Finding Patterns—All About That Base, Part 2.pdf
- Guided Notes (Teacher Guide and Model Notes)—All About That Base, Part 2.docx
- Guided Notes (Teacher Guide and Model Notes)—All About That Base, Part 2.pdf
- Guided Notes—All About That Base, Part 2 - Spanish.docx
- Guided Notes—All About That Base, Part 2 - Spanish.pdf
- Guided Notes—All About That Base, Part 2.docx
- Guided Notes—All About That Base, Part 2.pdf
- Lesson Slides—All About That Base, Part 2.pptx

Materials

- Lesson Slides (attached)
- Finding Patterns handout (attached; one per pair; printed front only)
- Guided Notes handout (attached; one per student; printed front only)
- Guided Notes (Teacher Guide and Model Notes) (attached; for teacher use)
- Applying Properties handout (attached; one per student; printed front only)
- Pencils
- Paper
- Scientific calculators (optional)

5 minutes

Engage

Display **slide 3** from the attached **Lesson Slides**. As students walk into class, ask them to answer the Bell Ringer question on a piece of notebook paper or elsewhere if you have a classroom norm for bell work.

After students have had a chance to answer this question, go to **slide 4** so students can check their work. Use this time to address any misconceptions that still exist from the previous lesson: "All About That Base, Part 1."

Go to **slide 5** to share the lesson's essential question with students. Go to **slide 6** to share the lesson's learning objectives. Review each of these with students to the extent you feel necessary.

20 minutes

Explore

Ask students to find partners or assign student pairs. Display **slide 7** and pass out the attached **Finding Patterns** handout to each pair of students.

Give students time to find Pattern 1. Then, go to **slide 8**. Once students see the pattern on the slide, ask for volunteers to explain the pattern in their own words. Hearing the pattern explained in more than one way can help other students better understand and more easily approach Pattern 2.

After sufficient student explanation of Pattern 1, give students time to find Pattern 2. Then, go to **slide 9** and repeat the process above.

After sufficient student explanation of Pattern 2, give students time to find Pattern 3. Then, go to **slide 10** and repeat the process above.

Teacher's Note: Guiding the Activity

If students struggle with the patterns, consider allowing time for student pairs to work before bringing everyone back together to find the patterns as a class. Ask students to share their thinking so far. Having a whole-class discussion adds more ideas to the conversation and often helps students find the pattern more quickly.

Students might struggle to find Pattern 3 in particular, as it is quite a bit more challenging than Patterns 1 and 2. Allow students to have some healthy struggle with this pattern. Then, give the following hints one at a time as needed until students are able to find the pattern:

1. Remember that your observed pattern must work for all three tables in that section of the handout.
2. What is different about this third set of tables? If it is that different, then you need a different approach. Try to take a step back and look at the pattern differently from how you approached Patterns 1 and 2.
3. What have you tried that almost worked?

Students often notice that Pattern 3 is a lot like Pattern 1 until they look at the last table: The product of the first two inputs results in the last input, except in that last table. Once students reach this point, continue with the following guiding questions:

4. Is there any operation similar to multiplication that yields very similar results? (exponents/powers)
5. Could you rewrite what is in the logarithm as a number to a power?

If needed, walk students through the patterns. Promote a healthy struggle, but do not let students get to the point of frustration and quitting.

20 minutes

Explain

Pass out the attached **Guided Notes** handout to each student. Transition through **slides 11–12** to go over the properties and their verbal descriptions with the class.

Using the hidden **slide 13** for quick reference, walk students through the proof of the change of base formula. Engage students by having them identify each property used during the proof.

Optional Activity

If time allows, have students use the change of base formula to get a decimal approximation of the answer to the bell ringer from the Engage portion of this lesson.

Display **slide 14** and complete the Guided Notes handout as a class.

Teacher's Note: Guiding the Activity

See the attached **Guided Notes (Teacher Guide and Model Notes)** document for detailed explanations of the proof and the example problems. Use this document for additional support and recommendations as you walk students through the Guided Notes.

Have students add their completed Guided Notes to their math notebooks if that is a classroom norm.

Teacher's Note: Vocabulary

When applying the exponential function as the inverse of a logarithm, students might be unsure of how to articulate what is happening. Help students strengthen their academic vocabulary by using phrases like "exponentiate both sides with base b" or "apply the exponential function with base b to both sides" or "raise both sides as the power with base b." These phrases mirror the way we describe "taking the logarithm base b of both sides" of an equation. Challenge students to use this academic language when they discuss their work.

20 minutes

Extend

Display **slide 15** and pass out the attached **Applying Properties** handout to each student. In pairs, have students use the Pass the Problem strategy to solve the given logarithmic equations from the Solving portion of the handout.

Have each pair of students get out a piece of notebook paper to show their shared work. Explain the procedure as follows:

- For question 1, student A writes the first step in the solving process.
- Student A then passes the paper to student B, who writes the next step.
- Students continue taking turns until the equation is completely solved.
- For question 2, student B starts instead and then passes the paper to student A, taking turns until solved.

As student pairs finish question 1, transition through **slides 16–17** so students can check their work and ask questions if needed.

As student pairs complete the remaining questions, transition through **slides 18–23**, again allowing students time to check their work and ask questions.

Go to **slide 24** and direct students' attention to the Evaluating portion of the handout. Explain to students that this is the time for them to work independently. Have each student evaluate the logarithmic expressions using the given information on the handout.

Collect the Applying Properties handout and use student responses to check for misunderstandings.

Teacher's Note: Differentiation

Some students may finish the Evaluating portion of the Applying Properties handout quickly; challenge these students to find the answer in more than one way using the properties of logarithms.

The Lesson Slides show two approaches for each problem. If reviewing the results as a class, consider asking the class if anyone evaluated any of the expressions differently from what is presented on the slides.

Optional Slides

After collecting the Applying Properties handout, if time allows, unhide and display **slides 25–27** to review how the properties of logarithms are used to evaluate logarithmic expressions.

15 minutes

Evaluate

Use the Justified True or False strategy to see which common misconceptions students may still have. Display **slide 28** and have students work with their partners to determine if each statement is true or false with justification.

Go to **slide 29** and ask for a volunteer to share what they and their partner decided for the first statement. Be sure to ask the student to justify their response. Click the slide to animate the answer for question 1.

Repeat this for the remaining statements, clicking again each time to display the answer to each statement.

Resources

- K20 Center. (n.d.). Bell Ringers and Exit Tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). Justified True or False. Strategies. https://learn.k20center.ou.edu/strategy/174
- K20 Center. (n.d.). Pass the Problem. Strategies. https://learn.k20center.ou.edu/strategy/151
- K20 Center. (n.d.). Desmos Classroom. Tech tools. https://learn.k20center.ou.edu/tech-tool/1081