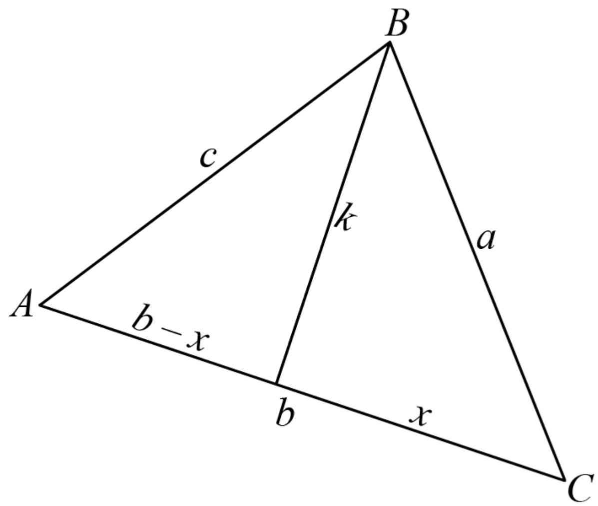
PROOF PROCESS

To develop the law of cosines, begin with ΔABC. From vertex B, altitude k is drawn and separates side b into segments $b-x$ and x.



1) Why can the segments be represented in this way?

2) The altitude separates ΔABC into two right triangles. Use the Pythagorean theorem to write two equations, one relating $b-x$, c, and k, and another relating a, k, and x.

3) Notice that both equations contain k^2.

   a) Why?

   b) Solve each equation for k^2.

4) Since both of the equations in Question 3 are equal to k^2, they can be set equal to each other.

   a) Why is this true?

   b) Set the equations equal to each other to form a new equation.

5) Notice that the equation in Question 4 involves x. However, x is not a side of ΔABC. Attempt to rewrite the equation in Question 4 so that it does not include x. Hint, begin by expanding the quantity $(b-x)^2$.

6) Now solve the equation for c^2.

7) The equation still involves x.

   a) To eliminate it from the equation, write an equivalent expression for x involving both $\cos(C)$ and x.

   b) Why use $\cos(C)$?

8) Solve the equation from Question 7 for x.

   a) Why solve for x?

9) Substitute the equivalent expression for x into the equation from Question 6 and simplify. The resulting equation contains only sides and angles of ΔABC. This equation is called the **Law of Cosines**.