Law of Cosines

Triangulation and Mapping

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Grade Level	11th – 12th Grade	**Time Frame**	80-90 minutes
Subject	Mathematics	**Duration**	1-2 class period(s)
Course	Precalculus		

Essential Question

How can indirect measurements help calculate unknown distances?

Summary

This is a lesson for Law of Cosines, which also connects Law of Sines and Law of Cosines to map triangulation. Students will receive a brief introduction to Trig Pillars: a tool used in the United Kingdom in the mid-20th century to help with mapmaking. Then, they will go through an informal proof of the Law of Cosines. Students will apply what they have learned to calculate distances across a school map. Law of Sines is a prerequisite for this lesson.

Snapshot

Engage

Students watch a quick video about Trig Pillars in the United Kingdom.

Explore

Students work through an informal proof of Law of Cosines.

Explain

Students generalize the Law of Cosines and formalize their understanding through class discussion.

Extend

Students practice map triangulation on a school map.

Evaluate

Students reflect on their learning and compare Law of Sines and Law of Cosines.

Attachments

- Calculating-Distance-Law-of-Cosines - Spanish.docx
- Calculating-Distance-Law-of-Cosines - Spanish.pdf
- Calculating-Distance-Law-of-Cosines.docx
- Calculating-Distance-Law-of-Cosines.pdf
- Calculating-Distance-Sample-Responses-Law-of-Cosines.pdf
- Lesson-Slides-Law-of-Cosines.pptx
- Proof-Process-Law-of-Cosines - Spanish.docx
- Proof-Process-Law-of-Cosines - Spanish.pdf
- Proof-Process-Law-of-Cosines.docx
- Proof-Process-Law-of-Cosines.pdf
- School-Map-Law-of-Cosines.pdf

Materials

- Lesson Slides (attached)
- Proof Process handout (attached; one per student; printed front/back)
- School Map handout (attached; one per student; printed front only)
- Calculating Distance handout (attached; one per student; printed front only)
- Calculating Distance (Sample Responses) document (attached; for teacher use)
- Protractors
- Straightedges
- Scientific calculators

10 minutes

Engage

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential question. **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Display **slide 5** and show students The Concrete Pillars On Top Of British Hills: Trig Points video, which gives students a brief overview of Triangulation Pillars, also known as Trig Points/Pillars, that are throughout the United Kingdom. These pillars were constructed and used during the mid-20th century by the Ordinance Survey to retriangulate the United Kingdom.

Embedded video
https://youtube.com/watch?v=VxBlyZAl5MQ

Show **slides 6–7**, which provide a bit more background information about Trig Pillars and the purpose of triangulation in calculating distances. Tell students that today they will learn how to use trigonometry to determine unknown distances.

15 minutes

Explore

Display **slide 8** and pass out the **Proof Process** handout to each student. Allow them some time to work through the steps on their own. After about 7 minutes of working on their own, show **slide 9** and have them share their answers with a partner and compare their progress using the Inverted Pyramid strategy. If their answers are not the same, give them a few minutes to talk through the differences and decide which answers they can agree to.

Teacher's Note: Not a Formal Proof

The questions are designed to flow from something that students know, the Pythagorean theorem, toward something that they do not know, the Law of Cosines. This is a quick way to justify why the Law of Cosines works and why adjacent sides and the contained angle are needed to use the Law of Cosines. Use the hidden **slides 10–11** for reference, if needed.

The handout is not set up as a formal proof, but as an extension to increase the difficulty, that would be an excellent parallel add-in prompt.

Teacher's Note: Proof Process Source

This activity is adapted from a lesson through the National Council of Teachers of Mathematics.

Johnson, H. L. (n.d.). Law of Cosines: Lesson 2. NCTM. https://illuminations.nctm.org/lesson.aspx?id=2441

15 minutes

Explain

Once students compare with a partner, continue the Inverted Pyramid by having a whole-group discussion.

Show **slide 12**, displaying the Law of Cosines, the result to the Proof Process handout: $c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$. Tell students that this should be their final result. Ask them, "How did you do?" After a few answers, ask, "If you got it wrong, where did you go wrong?" Once again, allow students time to discuss, share, and fix their work.

Then, direct students to look at both the correct equation and the triangle diagram from the Explore activity. Ask students what specific components are needed to make Law of Cosines valid or usable.

Teacher's Note: Guiding the Activity

Some students may catch on to the answer to that prompt quickly, but it is a big leap to go from one equation to the others when it is not explicitly told what is needed. Try not to just tell students that what is needed is two adjacent sides and the contained angle. Give hints like, "If the equation is solved for b, what do we need to know?" or, "How would that change if we wanted to solve for a instead?" And persevere; students will remember more when they work through it rather than when you give them the answer.

Finally, based on their ideas, prompt students to write equations solved for the other side lengths, a and b. Give students about 3 minutes to write out the equations, and then show them the correct answers on **slide 13**. Give them a moment to correct as needed. Before moving on, make sure that students have made the connection between the adjacent sides and the contained angle being the needed information to use the Law of Cosines.

35 minutes

Extend

Teacher's Note: Make It Your Own

This activity has directions that correspond to the map provided with this lesson. You can use this, but if you want a greater impact, use your own school's map and adjust the directions to fit your school. It will take more work, but it will have more value for your students.

Show **slide 14** and pass out a **School Map** handout and a **Calculating Distance** handout to each student.

Since students use each answer in their following calculations, show **slides 15–16** to make sure that students understand steps 1–2 of the activity and start the activity correctly. Then have students work independently to complete the activity. It is a productive challenge for them to try the activity independently, but if they struggle to do the activity on their own, have them work in pairs. If students follow the prompts correctly, then they will construct triangles on the school map, using a straightedge and protractor, and will utilize Law of Sines or Law of Cosines to calculate unknown distances on the school map. Use the hidden **slide 17** for quick reference regarding students' accuracy as you monitor the activity. Remember that there will be variety in your students' results. See the **Calculating Distance (Samples Responses)** document for more guidance.

Teacher's Note: Raise the Stakes

Depending on the personality of your class, a little healthy competition may be appropriate. Students' angles, measurements, etc., can get more and more off as the activity progresses if they are not accurate and precise in their measurements and drawings. To give them an incentive to use that level of precision needed, make it a contest. Whoever gets the closest to your key wins! Prizes do not have to be big. For some classes, even being heralded as the winner would be prize enough. If you are bold, consider allowing negotiation over what the prize could be.

Still recommend that students:

1. Round their angle measurements to the nearest degree.
2. Write down the rounded side lengths but use the results from the calculator to keep more decimal places and reduce rounding errors.

Teacher's Note: Calculating Distance Source

This activity is adapted from an activity by Jonathan Newman.

Newman, J. (2013). Law of Sines/Cosines "Mapquest" [Blog post]. Hilbert's Hotel. https://hilbertshotel.wordpress.com/2013/01/10/law-of-sinescosines-mapquest/

5 minutes

Evaluate

Show **slide 18** and instruct students to turn over their Calculating Distance handout to answer the questions on the slide on the back of their handout.

- In which situation(s) did you use the Law of Sines?
- In which situation(s) did you use the Law of Cosines?
- In which situation(s) could you have used the Law of Cosines? In which situation(s) could you have not? Justify your answers.
- What information could be used to determine if the Law of Sines or Law of Cosines is the most appropriate for the situation?

Collect the Calculating Distance handout to assess student learning. Look at their responses to check that students understand that to find the length of the side of a triangle, they need two angles and one opposite side to use the Law of Sines, and they need two sides and the included angle to use the Law of Cosines.

If time allows, have a class conversation about this distinction. In general, the Law of Cosines is best for finding the length of the third side of a triangle when the other two side lengths and included angle are known. The Law of Sines is best used to find the length of the third side of a triangle when two angles and one side length is given or to find the third side length when two sides and the non-included angle are known. Both formulas can be used to find unknown angles as well. Students at this point should start to be able to make the decision of which formula is most appropriate for a given situation. If students struggle with identifying the right formula to use, consider giving students a series of problems where the task is not to perform the calculations to find the missing information but to simply identify the right tool to complete the task. When doing this, be sure to have students justify why they picked Law of Sines or Cosines; this will help reinforce when to use each formula.

Resources

- Johnson, H. L. (n.d.). Law of Cosines: Lesson 2. NCTM. https://illuminations.nctm.org/lesson.aspx?id=2441
- K20 Center. (n.d.). Inverted pyramid. Strategies. https://learn.k20center.ou.edu/strategy/173
- Newman, J. (2013). Law of Sines/Cosines "Mapquest" [Blog post]. Hilbert's Hotel. https://hilbertshotel.wordpress.com/2013/01/10/law-of-sinescosines-mapquest/
- Scott, T. (2014, December 8). The concrete pillars on top of British hills: Trig points [Video]. YouTube. https://www.youtube.com/watch?v=VxBlyZAl5MQ
- Staziker, T. (2016, February 19). Mountain Trig Point Summit [Photograph]. Pixabay. https://pixabay.com/photos/mountain-trig-point-summit-1519813/