Matrix Operations

Matrices in Computer Graphics

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Grade Level	10th – 11th Grade	**Time Frame**	90-130 minutes
Subject	Mathematics	**Duration**	2-3 class periods
Course	Algebra 2		

Essential Question

How do we use matrices?

Summary

In this lesson, students will discover how matrices are used in computer graphics. Students will learn how to perform the matrix operations of addition, subtraction, multiplication, and scalar multiplication. They will then learn how multiplying matrices relates to transformations and, in turn, animation of computer graphics.

Snapshot

Engage 1

Students watch a video about the relationship between computer graphics and matrices.

Explore

Students find the connection between data and matrices.

Explain 1

Students start guided notes with the class and formalize their understanding of adding and subtracting matrices and scalar multiplication.

Extend 1

Students apply what they have learned to solve equations involving matrix operations.

Engage 2

Students watch a more in-depth video about the relationship between computer graphics and matrices.

Explain 2

Students complete guided notes with the class and formalize their understanding of multiplying matrices.

Extend 2

Students apply what they have learned to transformation matrices.

Evaluate

Students demonstrate their understanding of the criteria needed to perform a matrix operation.

Standards

Oklahoma Academic Standards for Mathematics (Grades 9, 10, 11, 12)

A2.N.1.3: Use matrices to organize and represent data. Identify the order (dimension) of a matrix, add and subtract matrices of appropriate dimensions, and multiply a matrix by a scalar to create a new matrix to solve problems.

Attachments

- Exit Ticket—Matrix Operations - Spanish.docx
- Exit Ticket—Matrix Operations - Spanish.pdf
- Exit Ticket—Matrix Operations.docx
- Exit Ticket—Matrix Operations.pdf
- Guided Notes—Matrix Operations - Spanish.docx
- Guided Notes—Matrix Operations - Spanish.pdf
- Guided Notes—Matrix Operations.docx
- Guided Notes—Matrix Operations.pdf
- Lesson Slides—Matrix Operations.pptx
- Missing Information—Matrix Operations - Spanish.docx
- Missing Information—Matrix Operations - Spanish.pdf
- Missing Information—Matrix Operations.docx
- Missing Information—Matrix Operations.pdf
- Problem Solved—Matrix Operations - Spanish.docx
- Problem Solved—Matrix Operations - Spanish.pdf
- Problem Solved—Matrix Operations.docx
- Problem Solved—Matrix Operations.pdf
- Transformation Cards—Matrix Operations - Spanish.docx
- Transformation Cards—Matrix Operations - Spanish.pdf
- Transformation Cards—Matrix Operations.docx
- Transformation Cards—Matrix Operations.pdf
- Transformation Matrices—Matrix Operations - Spanish.docx
- Transformation Matrices—Matrix Operations - Spanish.pdf
- Transformation Matrices—Matrix Operations.docx
- Transformation Matrices—Matrix Operations.pdf

Materials

- Lesson Slides (attached)
- Missing Information handout (attached; one per pair; printed front-only)
- Guided Notes handout (attached; one per student; printed front/back)
- Problem Solved handout (attached; one half-sheet per student; printed front-only)
- Transformation Matrices handout (attached; one per group of 4; printed front-only)
- Transformation Cards (attached; one set per group of 4; printed front-only)
- Exit Ticket handout (attached; one half-sheet per student; printed front-only)
- Pencils
- Paper
- Graph paper
- Student devices with internet access
- Scientific calculators (optional)

5 minutes

Engage 1

Teacher's Note: Lesson Order

The order of this lesson is as follows: Engage 1, Explore, Explain 1, Extend 1, Engage 2, Explain 2, Extend 2, Evaluate.

The lesson is structured this way for students to master matrix operations with increasing levels of complexity and to have the support needed to be successful.

Teacher's Note: Lesson Prep

Print and cut out the attached **Transformation Cards** (one copy per group of four students). Consider printing on cardstock or laminating, especially if you plan to reuse the cards.

Introduce the lesson using the attached **Lesson Slides**. **Slide 3** displays the lesson's essential question. **Slide 4** identifies the lesson's learning objectives. Review each of these with your class to the extent you feel necessary.

Display **slide 5** and show the video titled "Matrices in Computer Graphics."

Embedded video
https://youtube.com/watch?v=g8sMoZ2fgZw

Inform students they are going to learn some of the basics of moving figures using matrices during this lesson.

10 minutes

Explore

Instruct students to find a partner or assign students partners. Show **slide 6** and pass out the attached **Missing Information** handout to each pair of students. Instruct students to match the matrices (options A–D) with the given data (questions 1–4). Guide students to use their matches to answer questions 5–8 on the handout and find the unknown values.

When they are finished, show **slide 7** and allow students time to check their work. Ask for volunteers to share their reasoning.

15 minutes

Explain 1

Pass out copies of the attached **Guided Notes** handout and display **slide 8**. Complete the front side of the handout as a class.

After completing only the front side of the handout, direct students to set it aside. Students complete the back side later in the lesson.

5 minutes

Extend 1

Display **slide 9** and give each student a copy of the attached **Problem Solved** handout. Have students work in pairs to solve each equation using the matrix operations learned during the Explain 1 portion of the lesson. Encourage students to use their Guided Notes handout for reference.

When students finish, transition through **slides 10–11** and allow students time to check their work. If time allows, ask for volunteers to explain their process for solving each equation. Use student responses to identify any misconceptions.

10 minutes

Engage 2

Display **slide 12** and introduce the video "How Rendering Graphics Works in Games." As they watch, students will see more clearly that matrices are used in computer graphics. Prepare the class for the reflective question that they are asked to respond to after watching the video.

> *What did you learn from the video that you did not know?*

Embedded video

https://youtube.com/watch?v=cvcAjgMUPUA

Teacher's Note: Video Timing

Having students watch this video just before completing the back side of the Guided Notes handout (in the next portion of the lesson) will help to re-engage the class and capture their attention.

This video is still a surface-level overview of how matrices are used in computer graphics. Assure students that they do not need to take notes during the video or remember any of the specifics, as the purpose of this lesson is not for students to have a deep understanding of specifically how matrices are used, but rather to understand the fact that matrices are used in animating computer graphics and, most importantly, how to perform matrix operations.

Discuss students' responses to the reflective question: *What did you learn from the video that you did not know?* Ask for volunteers to share.

Transition into the next activity by telling the class that they are now going to learn how to perform matrix multiplication, which is different from scalar multiplication.

15 minutes

Explain 2

Show **slide 13**. Start completing the back side of the Guided Notes handout as a class by teaching students how to multiply matrices using the information at the top of the page and in example 3.

Before starting example 4, read the following to the class to introduce the example:

> *Sports are an excellent source of data. Think about all of the numbers used to describe a basketball player's performance, like their offense, defense, and rebounding. These data can be used to rank players. For example, EA Sports uses data for rankings in NBA 2K video games. But how do they do it? When processing that amount of data, computers are performing the calculations using matrices. They multiply the players' ratings by the weights of the categories to yield the overall player ranking.*

Complete example 4 as a class. Have students add their completed Guided Notes handouts to their math notebooks if that is a classroom norm.

Teacher's Note: ACT Prep

Students might encounter a matrix multiplication question on the ACT exam. These questions often involve multiplying a small matrix by another small matrix, like a 2x2 matrix times another 2x2 matrix. If a student knows how to perform this operation by hand, doing so is often faster than trying to input those values into a calculator to perform the operation.

20 minutes

Extend 2

Direct students to get into groups of four or assign students to groups. Show **slide 14** and pass out a copy of the attached **Transformation Matrices** handout and a set of **Transformation Cards** to each group. Direct students to read and follow the steps on the handout. Here students are to select a figure, select four transformation cards, and use matrix multiplication to transform their selected figure.

As students work, circulate around the room. Once you notice most students are on the back side of the handout, transition to **slide 15**. This slide helps clarify step 5 by showing students how to write their vertices as matrices to use for matrix multiplication.

Teacher's Note: Transformation Matrices

Transformations can be applied one at a time, which is what students often see in a geometry class. However, multiple transformations can be applied at once using matrix multiplication. The purpose here is for students to apply what they learned during the Explain 2 portion of the lesson and multiply matrices. Students do not need to find the relationship between the given transformation and the given matrix.

Teacher's Note: Shearing Transformation

Horizontal shearing is where the y-coordinates stay the same, while the x-coordinates that are not along the x-axis change. Vertical shearing is where the x-coordinates stay the same, while the y-coordinates that are not along the y-axis change. This is not a transformation that students typically see in a high school geometry course.

After students finish multiplying their transformation matrix, instruct them to use the graph paper to draw their original figure and their new figure. As students finish their sketches, share **slide 16** and direct them to use their devices to access the GeoGebra activity link. Students should click the transformation buttons in the order in which they want them applied. Remind students that this order is the opposite order of how they multiplied their matrices.

Teacher's Note: Activity Inspiration

This activity was inspired by this CGI matrix multiplication activity created by Dr. Cynthia Huffman from Pittsburg State University.

10 minutes

Evaluate

Use the Exit Ticket strategy to individually assess what students have learned from the lesson. Go to **slide 17** and pass out copies of the attached **Exit Ticket** handout. Students are asked to perform the possible indicated matrix operations. If the operation is not possible, they are to write *impossible*. Use student responses to identify what misconceptions still exist and guide further instruction.

Optional Slides

Unhide and show **slides 18–19** if you would like students to check their work.

Alternative Pacing

This Exit Ticket can be completed in class or assigned for students to turn in when they come to class the next day. Reviewing matrix operations could also be done as bell work the following day. Consider giving the Exit Ticket handout as homework, and then starting the next day with a 3–5 minute review using slides 18–19.

Resources

- Bitbinge. (2017, December 19). Kingfisher [Illustration]. Pixabay. https://pixabay.com/illustrations/bird-halcyon-kingfisher-eisvogel-3027224/
- Huffman, C. J. (2019). CGI matrix multiplication activity. Open Educational Resources - Math. Pittsburg State University Library Services. https://digitalcommons.pittstate.edu/oer-math/16
- K20 Center. (n.d.). Bell ringers and exit tickets. Strategies. https://learn.k20center.ou.edu/strategy/125
- K20 Center. (n.d.). GeoGebra. Tech Tools. https://learn.k20center.ou.edu/tech-tool/2352
- K20 Center. (2022, July 21). Matrices in computer graphics [Video]. YouTube. https://youtu.be/g8sMoZ2fgZw
- McLaughlin, A. [TheHappieCat]. (2015, July 22). How rendering graphics works in games! [Video]. YouTube. https://youtu.be/cvcAjgMUPUA