

# HOW LONG IS A BORDER?

## Exploring Convergent and Divergent Series

**Background:** “How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension” was a paper written by mathematician Benoît Mandelbrot, first published in *Science* in 1967. In this paper, Mandelbrot discusses self-similar curves that have a dimension between 1 and 2, called the Hausdorff dimension. These curves are examples of fractals. One of the interesting ideas that came out of his paper is that you can measure a coastline with different length “yardsticks,” and you’ll get different measurements each time.

**Questions to Consider:** How long is a border and how can you measure it? Why do we have borders? What paradoxes do you see in the outcomes of perimeter and area? Is there a right way to measure borders and/or coastlines?

**What you’ll need:** Measuring sticks, pen or pencil, a map of a coastline, and some knowledge of convergent and divergent series.

**Some brief notes:** Discuss at your table how to tell if a series converges or diverges. Also talk about how limits play a role in convergent/divergent series.

1. What does it mean when a series converges? Give an example of a convergent series.
2. What does it mean when a series diverges? Give an example of a divergent series.
3. Use your different measuring sticks, measure the coastline three times with the three different sized sticks. Start with the longest and end with the shortest. Note: If the stick doesn’t fit neatly, just go around the “bumpy” parts of the graphs. Do not use fractional measurements.

Record your findings:

4. What do you notice about the three measurements? Are they getting longer or shorter? Explain.

5. If the measurements of the perimeter were written in a series, would you say the series is convergent or divergent? How do you know? Explain.

6. Although we did not measure the area of Oklahoma, would you say the area is converging or diverging? How do you know? Explain.

7. What paradoxes do you see?

8. How does what we measure influence how we measure? How does how we measure influence what we measure?

**Yardsticks to cut out:**

Yardstick 1

Length: 1.5 inch



Yardstick 2

Length: 0.75 inch



Yardstick 3

Length: 0.25 inch



