

# Thinking Outside the Equation for High School Math Teachers: Learning Activity 2 of 6 - Spatial Reasoning

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**Time Frame** 60-90 session(s)

## Essential Question(s)

- How does one engage in non-routine mathematical tasks?
- How can non-routine tasks promote spatial reasoning?

## Summary

Mathematics is sometimes perceived as one directional, but it doesn't have to be. In this interactive professional development institute, participants will reflect upon what 'math' really is, how it looks and feels, and how intentional tasks can enhance overall understanding and critical thinking. Participants will experience mathematical concepts via non-routine tasks that reach beyond procedural 'steps' and shift into mathematical understanding that builds a flexible mindset for teaching and learning. Participants will construct instructional tasks for their classroom and return to reflect over the success of their task and how to move forward growing as teachers of mathematics.

## Learning Goals

- Develop participants' understanding of authentic researched-based mathematical practices.
- Explore and develop non-routine math tasks that correlate to appropriate grade levels.
- Demonstrate the incorporation of strategies and instructional tasks that foster a growth-mindset for flexible mathematics.
- Deepen participants' understanding of number sense and mathematical concepts through interactive, non-routine tasks, and student inquiry.

## Attachments

- [Doodle Pictures.docx](#)
- [Thinking Outside the Equation\\_High School PDI.pptx](#)
- [Unit Origami Handout.pdf](#)

## Materials

- Doodles
- 8.5 by 11 copy paper cut to 8.5 by 8.5
- Pen/Pencil
- Scratch Paper

# Engage

## Preparation

First, make copies of the Doodle Pictures in the Doodle Pictures handout. Cut out pictures and set aside for participants to choose from. If you would like to re-use these for future activities, I suggest laminating them. Additionally, please print a copy of the Unit Origami Handout for each participant.

## Set-Up

For this PD, it is best if participants are working in groups. While some activities will be done individually, having peers nearby for discussion is paramount. Have square sheets of copy paper ready to go for this PD. Each participant will need at least three sheets (you can distribute them or you can have them choose their papers when it is time for the activity). Square sheets can be different colors if you choose (they make for prettier designs).

To kick things off, participants will engage in a short activity involving Doodles. These are pictures with interesting shapes on them, many of which contain interesting shapes and lines.

Set out the Doodle Pictures on a table. Ask participants to come up to the table to choose a Doodle that they like, find interesting, or that "speaks to them."

Upon returning to their tables, ask participants to make mental notes about what they see in the Doodle and to rationalize why they chose it.

Depending on the size of your PD, ask participants to share why they chose their Doodle to the whole group (less than 15 participants) or to their table-mates (15 or more participants).

After participants have shared, ask them to discuss at their groups the geometry they see within the Doodle.

# Explore

## Unit Origami

The purpose of exploring unit origami is to engage participants in a hands-on activity that allows them to explore spatial reasoning and pattern recognition. This exploration is rooted in Burgeil's *Unit Origami: Star Building on Unit Deltahedra*. Some aspects of the lesson have been modified.

## Set-Up

Before embarking on this non-routine task, ensure that all participants have access to at least three, square 8.5 by 8.5 paper and the Unit Origami Handout.

### Part 1: Constructing and Analyzing One Star Building Unit

Using the Unit Origami Handout, follow the steps on pages 586-587. This will allow you to construct one star building unit.

After construction, unfold what you've created. Yes, unfold it after all your hard work...

Answer the following questions and discuss with your tablemates:

- What do you notice?
- What geometric shapes and/or structures are present?
- What symmetries do you see?
- How many parallel lines exist? Transversals? Perpendicular lines?

### Part 2: Constructing a Cube

Constructing a cube is simply a puzzle. Once you have either reassembled your deconstructed unit or rebuild a new one, you can build a cube with a little more work.

Make **six** star building units in the exact same fashion as in part 1

## Orientation Is Key

Be sure to make all six in the same way. What I mean is this: if you orient your paper the wrong way and create left-hand folds instead of right-hand folds (or vice-versa), then the components of the stars will not fit together correctly.

Begin fitting the units together. This takes some time and practice, but the goal is to use the flaps and "arms" from the star building unit to fit together to form a cube.

## Explain

Still working in groups, pose the following question to participants in a [Think-Pair-Share](#): What did you learn/notice about the activity?

**Think:** Ask participants to think about the question in terms of mathematical content and application to their classrooms for about 1 minute.

**Pair:** Have participants share with one person within their group. Give each participant about 2 minutes to share.

**Share:** Solicit a handful of responses from those willing to share with the whole group. Respond to each participant or open the floor to others to share their thinking.

## Extend

Metacognitive Reflection: Ask participants to take a few minutes to silently reflect and write about TWO of the following questions:

- Thinking about growth mindset in your practice, what things would you need to change and how would you change them in order to implement this task?
- What mathematics are students being engaged in?
- In what ways was this task low floor/high ceiling?What was the task?
- How is it meaningful to students?
- How can you implement this into your class with your students?
- What standards were addressed within this activity for my grade level?
- Where do you see students' potentially growing in their spatial reasoning skills?

# Evaluate

*This particular PD did not have an evaluate section for this learning activity. However, feel free to use the following instructional strategy as a way to evaluate participant learning:*

[3-2-1](#)

What are **three** things you are thinking about now that you have finished this learning activity?

What are **two** things you could try in your classroom?

What is **one** question or concern you have?

## Follow-up Activities

Parking Lot: As participants move through this learning activity and those that follow, have them jot down questions, comments, and concerns they may have regarding what they are learning, what is being presented, and what they are thinking. These can be written on a sticky note and placed on a designated area in the room labeled Parking Lot.

Periodically remind participants they can post sticky notes here with their questions, comments, and/or concerns.

Before and after breaks are good times to address quandaries posted to the parking lot.



## Research Rationale

Mathematics education has a rich and complex history, shaped largely by major educational reform initiatives. At the turn of the twentieth century, mathematics classrooms were often seen as learning laboratories which incorporated hands-on lessons and inquiry-based learning scenarios similar to what one might see in a science lab (Kilpatrick, 2014). Influential educators, like Dewey, were at the forefront, implementing constructivist pedagogies hinged on students constructing knowledge and making meaning for themselves (Dewey, 1950; Kilpatrick, 2014; Schiro, 2013). These pedagogies were in stark contrast to traditional, lecture-based disseminations of information during this period in history. Throughout the decades following, political agendas and industrial demands in the US aimed at producing more mathematicians and scientists, along with international competition to be the best in mathematics, have retrograded pedagogical strategies to less progressive eras in history. Major educational reforms driven by federal initiatives like "A Nation at Risk," "No Child Left Behind," and "Race to the Top" have placed major emphasis on efficiency in mathematics education, thus promoting a factory model for schools (Kilpatrick, 2014; Schiro, 2013; Teitelbaum, 2014). As a result, teaching and learning today looks very similar to how it did in the nineteenth century, but with more emphasis on holding teachers accountable to teaching prescribed standards. So, it is no surprise then that the US has consistently fallen behind other countries in terms of mathematics achievement (Kilpatrick, 2014; Teitelbaum, 2014). Despite pushes for efficiency in mathematics education, educators like Boaler (2016) have been essential to the survival of progressive mathematics education. She and others view mathematics learning as less of a gift and more of a learned discipline. She has championed for pushes to foster growth mindset in mathematics teaching and learning, which is where this professional development is rooted. Boaler (2015) shares that students' mindset about mathematics greatly shapes their beliefs about themselves as learners and impacts their confidence in constructing mathematical knowledge. There exist two types of mindsets: fixed and growth. People who have fixed mindsets about mathematics tend to view themselves as being able to reach a limit for how much they can understand. On the other hand, those who have a growth mindset are more inclined to persevere through mistakes and tend to have positive beliefs about themselves as mathematics learners as they essentially have endless potential to learn (Boaler, 2015; Yeager & Dweck, 2012). Furthermore, mindset tends to be infectious. Students whose teacher has a growth mindset about mathematics were shown to have higher achievements and better mathematical performance, even those students who were tracked into lower-level courses (Dweck, 2010).

## Resources

- Burgiel, H. (2015). Unit Origami: Star-building on Deltahedra
- Boaler, J. (2015). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching. Jossey Bass: San Francisco, CA.
- Boaler, J. (2016). The importance of teaching in the promotion of open and equitable mathematics environments. In E. A. Silver & P. A. Kenney (Eds.), More lessons learning from research: Helping all students understand important mathematics Volume 2 (19-26). Reston, VA: National Council of Teachers of Mathematics.
- Dewey, J. (1950). The child and the curriculum (25th ed.). Chicago, IL: The University of Chicago Press.
- Dweck, C. (2010). Mind-sets and equitable education. *Principal Leadership*, 10(5), 26-29.
- Kilpatrick, J. (2014). Mathematics education in the United States and Canada. In A. Karp & G Schubring (Eds.), *Handbook on the History of Mathematics Education* (323-334). New York, NY: Springer.
- Schiro, M. (2013). *Curriculum theory: Conflicting visions and enduring concerns* (2nd ed.). Thousand Oaks, CA: Sage Publications.
- Teitelbaum, M. (2014). *Falling behind?: Boom, bust, and the global race for scientific talent*. Princeton, NJ: Princeton University Press.
- Yeager, D. & Dweck, C. (2012). Mindsets that promote resilience: When students believe that personal characteristics can be developed. *Educational Psychologist*, 47(4), 302-314.